Discussion

The purpose of this study is to determine the effect of luminosity distance on gravitational wave properties. The study involved obtaining the peak amplitude and frequency of gravitational waves from the inspiral of black holes and neutron stars.

The results for frequency were consistent with the hypothesis that frequency decreases with distance due to cosmic expansion.

The frequency of a gravitational wave was shown to decrease with distance in a polynomial relationship (figure 10). A model for a 200Hz gravitational wave was created using a derivation from Hubble's law. The value of Hubble's constant used was $67.4\ km\ s^{-1}/Mpc$ from measurements of the cosmic microwave background (Yun Chen, 2017), and the resulting plot showed a polynomial relationship with an almost identical shape to the frequency data. Given these similarities, the frequency of a gravitational wave thus decreases with distance as predicted by cosmic expansion. The value of Hubble's constant has negligible effect on the shape of the model's curve, so no comment on its value can be made.

These results align with previous research on the luminosity distance – redshift relation, which showed a non-linear relationship between luminosity distance and redshift, and by extension a non-linear relationship between frequency and luminosity distance (Padmanabhan, 2005).

An estimation of Hubble's constant was determined using the redshift and luminosity distance of each event. The results returned an average constant of $73.9\ km\ s^{-1}Mpc^{-1}$, which is greater than the value of $67.4\ km\ s^{-1}Mpc^{-1}$ from measurements of the cosmic microwave background (Yun Chen, 2017). However, there are also many other competing measurements of the Hubble constant obtained using different methods. Some such measurements include $70.5\ km\ s^{-1}Mpc^{-1}$ from type 1a supernovae (Nandita Khetan, 2021), as well as $67.8\ km\ s^{-1}Mpc^{-1}$ from gravitational wave event GW170817 (Hsin-Yu Chen, 2018).

These discrepancies reveal that an exact value for the present-day Hubble constant is still unknown. The Hubble constant calculated in this study uses luminosity distance obtained from gravitational wave properties and redshift obtained from measurements of a star's spectral lines in a nearby galaxy. The length of galaxies reaches up to 1.2Mpc (Juan M. Uson, 1991), so there is possibly a difference between the actual redshift and the redshift measurement. Thus, resulting in uncertainties in the value of the Hubble constant estimated.

An accurate value of the Hubble constant is important as it allows the redshift and hence the emitted frequency of a gravitational wave to be predicted to a greater accuracy whilst not requiring any additional measurements. Earlier, the Hubble constant was used to create a model of frequency vs luminosity distance for a 200 Hz gravitational wave, although the value of the constant had minimal effect on the curve, any slight change in the emitted frequency predicted from this model has a significant effect on the properties of binary systems that are determined from the data. Thus, an accurate value of Hubble's constant is necessary to accurately determine the properties of binary systems, as well as other systems that produce gravitational waves.

The results for amplitude were consistent with the hypothesis that amplitude is inversely proportional to distance, however the exact rate of change appeared to be slower than expected.

The amplitude of a gravitational wave was shown to decrease with distance in an inverse relationship (figure 12). A model was created for a gravitational wave produced from a binary system with chirp mass 25.9 M_{sun} and GW frequency 130 Hz. The model showed a greater extent of curvature over the given range, indicating an overall faster rate of change of amplitude than the data.

A plot of amplitude over inverse distance (figure x) was created for both the LIGO data and model, and the resulting plot was a linear regression with a smaller gradient in the data (4332.7 compared to 6925.6), indicating that amplitude decays at a slower rate than expected.

To account for this, the effects of background noise on the measured amplitude was estimated. The average noise amplitude 1 – 2s after the event time was measured, then assuming noise destructively interfered with the gravitational wave signal, the actual amplitude was calculated as the sum of noise amplitude and measured peak amplitude.

The resulting plots showed that the amount of curvature increased (figure 14), and the gradient of the inverse distance graph increased (figure 15). As a result, the adjusted data more closely aligns with the predicted model, suggesting that background noise was the cause of the discrepancies between data and prediction. However, it is also possible for background noise to constructively interfere with the gravitational wave signal, so the adjusted data aligning with the model could be a coincidence. Therefore, we cannot be certain that noise was the cause of differences between data and prediction.

Other explanations for the difference between data and prediction include interference with stochastic gravitational wave backgrounds, or the production of gravitational waves from dark matter and dark energy.

A stochastic gravitational wave background is the relic gravitational waves from the early evolution of the universe (LIGO. SC, 2016). The effect of a stochastic gravitational wave is either; (1) two waves collide and interfere, resulting in a new waveform, (2) stochastic gravitational waves pass through detector at the same time as the event, effecting the measured amplitude. In both cases, the effect could be either an increase or decrease in measured amplitude. In case (1), events with greater luminosity distances would have a higher chance of interacting with stochastic gravitational waves, and if these interactions were constructive the measured amplitudes would be greater, and the amplitude - luminosity distance curve would flatten. In case (2), the stochastic gravitational wave would be background noise interfering with the signal, measurements of its phase and amplitude would allow us to predict its effect on the gravitational wave signal. However, the amplitude of the stochastic gravitational wave background is unknown, so therefore the influence of stochastic gravitational waves cannot be determined.

The other possibility is the production of gravitational waves from dark matter and dark energy, which make up 27% and 68% of the universe respectively (CERN, n.d.). In recent studies, dark matter is theorised to produce GWs via dark photon production (Namba, 2020), while dark energy is theorised to produce GWs via its decay (Weiner, 2021). As a gravitational wave passes through dark matter/energy, they produce gravitational waves which interfere with the event wave resulting in a new waveform. If this interference is constructive, events of greater luminosity distances would encounter more dark matter/energy, and thus have more interference, resulting in the amplitude – luminosity distance curve to flatten. For both types of GW production however, the interference could be destructive making it unable to account for the observations, and the theories have not been experimentally tested.

Further research into the properties of the stochastic gravitational wave background, as well as the production of gravitational waves from dark matter and dark energy is required to further understand the factors affecting the amplitude of gravitational waves.

Conclusion

The frequency of a gravitational wave was shown to decrease with distance in a polynomial relationship. The shape of the frequency – luminosity distance curve matched the shape of a predicted model, therefore the results are consistent with the hypothesis that frequency decreases with distance due to cosmic expansion. The value of Hubble's constant was estimated to be $73.9\ km\ s^{-1}/Mpc$, which is inconsistent with previous estimates of $67.4\ km\ s^{-1}/Mpc$, $67.8\ km\ s^{-1}/Mpc$ and $70.5\ km\ s^{-1}/Mpc$. These inconsistencies reveal that a value for the present-day Hubble constant is unknown, thus further research into more accurate methods for measuring Hubble's constant should be conducted. The amplitude of a gravitational wave was shown to decrease with distance in an inverse relationship, however the amplitude – luminosity distance curve was flatter than expected, suggesting that amplitude decreased at a slower rate than expected. It was postulated that this difference could be due to interference with stochastic gravitational wave backgrounds, or from gravitational production from dark matter and dark energy, however with no means to account for this, their effects on the measured amplitude cannot be determined. When the effect of noise (assuming destructive interference) was accounted for, the resulting curve was steeper, and more closely aligned with the model. Therefore, the difference between data and prediction is most likely due to background noise destructively interfering with the gravitational wave signal. Thus, the results are consistent with the hypothesis the amplitude is inversely proportional to luminosity distance.